An Inductance-Capacitance Oscillator of Unusual Frequency Stability*

J. K. CLAPP†, FELLOW, I.R.E.

Summary—An *L-C* oscillator having unusual frequency stability is described and briefly analyzed. The circuit is similar to the familiar Colpitts, with an *L-C* series circuit replacing the inductor. Such a circuit has been used as a piezoelectric oscillator, with the quartz crystal replacing the *L-C* series circuit, but the circuit does not seem to have been previously described as an *L-C* oscillator.

THE OSCILLATOR considered here has been in use for some years as a quartz-crystal oscillator.[1] Because of the inherent stability associated with the quartz crystal, not much attention has been paid to the possibilities of the circuit for use as an *L-C* oscillator. In such applications the circuit does not seem to have been described previously.

In the usual forms of pi-network oscillators, such as the Hartley and Colpitts, the plate and grid resistances of the tube are shunted across elements of the tuned circuit. Variations of these resistances, as by changes in electrode supply voltages, cause appreciable changes in the frequency of oscillation. In the Hartley circuit it is well known that tapping the tube across only a portion of the tuned-circuit inductance results in a substantial improvement in frequency stability. Such a circuit has been described by Lampkin.[2] This arrangement, indicated schematically in Fig. 1(a), has two practical disadvantages: first, it has a very strong tendency to generate spurious oscillations in the circuit formed by the tube capacitances and the tapped portion of the inductance; and second, the circuit is not too convenient for band-switching operation.

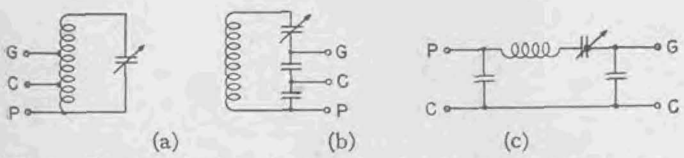

Fig. 1—Showing (a) tube tapped across part of inductive branch; (b) across part of capacitive branch; and (c) the circuit of (b) in filter representation.

The counterpart of this arrangement, with the tube tapped across a portion of the capacitive branch, is indicated in Fig. 1(b). With moderate care in keeping the connections to the tube short, there is practically no tendency toward spurious oscillation. In the configuration shown, only one point would have to be switched for multiband operation, but in practical circuits it is generally better to switch both ends of the inductor. When drawn in the form of a Colpitts circuit, the network appears as in Fig. 1(c), where it is evident that the low-pass (Colpitts) and high-pass (Hartley) arrangements have been replaced by a band-pass circuit.

A schematic diagram of a practical circuit is shown in Fig. 2. In effect, it is a grounded-plate arrangement, and no high d.c. voltages appear at any point of the tuned circuit. For convenience, one side of the tuning capacitor is grounded. The coupling capacitances C_1, C_2 are large compared to the tuning capacitance C_5, and are huge compared to the tube capacitances. The radio-frequency choke provides the cathode d.c. return path; preferably the choke should be capacitive at the operating frequency. Variation of the screen-grid voltage provides a convenient means of adjusting the amplitude of oscillation. In normal operation the coupling capacitances are made just as large as reliable operation will permit. Under such conditions the grid and plate swings are only a very few volts.

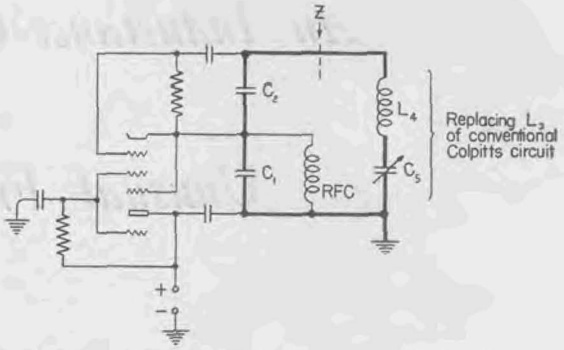

Fig. 2—Schematic diagram of circuit. Capacitances C_1 and C_2 are many times larger than those usually employed in a Colpitts oscillator.

It is informative to consider the circuits shown in Fig. 3. Starting from the simple circuit of Fig. 3(a), we find for the change in frequency caused by a change in tuning capacitance

$$\frac{df}{f} = -\frac{1}{2}\frac{dC_0}{C_0}. \qquad (1)$$

To fix ideas, let the circuit resonate at a frequency of 1 Mc., in which case C_0 might be, conveniently, 100 $\mu\mu$fd.; df/f then is $-dC_0/200$.

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† General Radio Company, Cambridge, Mass.

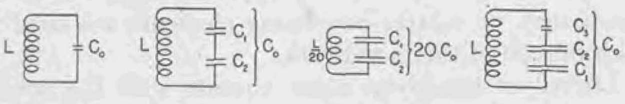
[1] For example, the General Radio Type-475-C oscillator, used in broadcast frequency monitors, which was introduced in 1940.

[2] G. F. Lampkin, "An improvement in constant-frequency oscillators," PROC. I.R.E., vol. 27, pp. 199–201; March, 1939.

In the form of a Colpitts oscillator, the circuit of Fig. 3(b) might have $C_1 = C_2 = 200$ $\mu\mu$fd. In this case, we will associate a change in capacitance with C_2, representing the input side of the tube. Then we have

$$\frac{df}{f} = -\frac{1}{2}\frac{C_1}{C_1+C_2}\frac{dC_2}{C_2} = -\frac{1}{2}\frac{C_0}{C_2}\frac{dC_2}{C_2}. \tag{2}$$

For the values given, $df/f = -dC_2/800$.



(a) (b) (c) (d)

Fig. 3—Illustrating circuit configurations which successively reduce the variation in resonant frequency caused by a given change in capacitance. The given change in capacitance is associated with C_0 in (a) and with C_2 in (b), (c), and (d).

Keeping the same circuit, but utilizing $C_1 = C_2 = 40$ C_0 in Fig. 3(c) with an inductance $L/20$, gives $df/f = -dC_2/16000$. This represents operation carried to the largest usable capacitances—a "High-C" circuit.

Turning now to the circuit of Fig. 3(d), representing Fig. 2, with $C_1 = C_2 = 40$ C_0 and $C_3 = 20$ $C_0/19$, the total capacitance is C_0. Then $df/f = -dC_2/320{,}000$, representing all of the improvement realized with "high-C" operation augmented by a factor due to the greatness of C_1, C_2 as compared with C_3. If we consider the frequency variation of Fig. 3(b), representing a conventional Colpitts oscillator, as unity, then the frequency variations of Fig. 3(b), (c), and (d) stand as 1, 1/20, 1/400, *for a given capacitance change.*

The circuit of Fig. 3(c) represents an improvement of up to twenty times or so over the conventional Colpitts, Fig. 3(b). In practice, this circuit would require a double variable tuning capacitance of large value. The circuit of Fig. 3(d) gives a further improvement of twenty times or so, using a single variable tuning capacitance of more usual value.

While the above development is by no means complete for determining the stability of frequency of an oscillator, it nevertheless indicates very well the relative improvement attainable with respect to changes in input capacitance of the tube as a result of temperature changes of tube structure or of changes in supply voltages.

using the circuit of Fig. 2. A simple approach is to divide the circuit at the dotted line and write the expression for the impedance Z seen looking into the circuit.

$$Z = Z_g + \frac{r_pZ_p}{r_p+Z_p} + \frac{\mu}{r_p}Z_g\frac{r_pZ_p}{r_p+Z_p} + Z_3. \tag{3}$$

Taking Z_g as r_g in parallel with X_2, Z_p as X_1, alone, separating reals and imaginaries and placing them equal to zero, we have, for a conventional Colpitts circuit,

$$-\frac{\mu}{r_p}X_1X_2 + \frac{X_1}{r_p}(X_3 - X_2) + \frac{X_2}{r_g}(X_3 - X_1) + R_3\left(1 - \frac{X_1X_2}{r_gr_p}\right) = 0 \tag{4}$$

$$-X_1\left(1+\frac{R_3}{r_p}\right) - X_2\left(1+\frac{R_3}{r_g}\right) + X_3\left(1 - \frac{X_1X_2}{r_gr_p}\right) = 0. \tag{5}$$

In (4) for reals, we can substitute with small error $(X_3 - X_2) = X_1$ and $(X_3 - X_1) = X_2$, and obtain

$$-\frac{\mu}{r_p}X_1X_2 + \frac{X_1^2}{r_p} + \frac{X_2^2}{r_g} + R_3 = 0 \tag{6}$$

where the first term represents the negative resistance developed in the circuit by the tube; the next two terms represent the equivalent series resistances of the tube resistances in parallel with the coupling capacitances; and the last term is the resistance of the inductor (neglecting the very small correction term).

In (5) for imaginaries, the terms in r_g are the ones causing a change in frequency with change in supply voltage. These are $X_1X_2X_3/r_gr_p$ and R_3X_2/r_g. With large reactances and low resistances, the first of these entirely overshadows the second. On reduction of the reactances to the lowest possible values, keeping the resistances of the tube as high as possible, however, the second becomes the predominant term.

An important analysis of the stability of this class of oscillators is given by Fair,[3] but the effects of the resistance of the inductor are not taken into account. Following Fair's method and including the coil resistance, we obtain

$$\frac{\delta\omega}{\delta V} = \frac{\dfrac{1}{\mu}\dfrac{\partial\mu}{\partial V}\left[\dfrac{X_1X_2X_3}{r_gr_p} + \dfrac{R_3X_2}{r_g}\right]}{\left[1 - \dfrac{X_1}{\mu X_2} - \dfrac{R_3r_p}{\mu X_1X_2}\right]\left[-\left(1+\dfrac{R_3}{r_p}\right)\dfrac{\partial X_1}{\partial\omega} - \left(1+\dfrac{R_3}{r_g}\right)\dfrac{\partial X_2}{\partial\omega} + \dfrac{\partial X_3}{\partial\omega}\right]} \tag{7}$$

for $\quad \mu = f_1(V), \quad r_g = f_2(a), \quad X_1 + X_2 + X_3 = f_3(\omega), \quad r_p = \text{constant}.$

A brief linear analysis is helpful in showing the improvement in stability of frequency which is possible

[3] I. E. Fair, "Piezoelectric crystals in oscillator circuits," *Bell Sys. Tech. Jour.*, vol. 24, pp. 161–215; April, 1945.

This is in the form given by Fair and differs from his result only by the terms in R_3.

The term $-R_3 r_p/\mu X_1 X_2$, in the first factor in the denominator, is the ratio of the coil resistance to the negative resistance developed in the circuit. From the equation for reals it is seen that $\mu X_1 X_2/r_p$ exceeds R_3 by only a small amount. The ratio is then near unity and the first factor reduces to $-X_1/\mu X_2$. The second factor in the denominator (neglecting the terms in R_3 which are small) is equal to $2L_3$. If we write X_3/Q_3 for R_3, (7) reduces to

$$\frac{\delta\omega}{\delta V} = \frac{\dfrac{1}{\mu}\dfrac{\partial\mu}{\partial V}\left[\dfrac{X_1 X_2 X_3}{r_g r_p} + \dfrac{X_2 X_3}{Q_3 r_g}\right]}{-\dfrac{X_1}{\mu X_2}[2L_3]} \tag{8}$$

$$= -\frac{X_2}{X_1}\frac{\partial\mu}{\partial V}\left[\frac{1}{2\omega C_1 C_2 r_g r_p} + \frac{1}{2Q_3 C_2 r_g}\right] \tag{9}$$

$$= -\frac{\partial\mu}{\partial V}\left[\frac{1}{2\omega n^2 C_0^2 r_g r_p} + \frac{1}{2Q_3 n C_0 r_g}\right] \tag{10}$$

for $\quad C_1 = C_2 = nC_0.$

In the ordinary Colpitts circuit the first term predominates. If the tuning capacitances C_1, C_2 are increased as much as possible and maintain oscillation ($C_1 = C_2$, L_3' adjusted for same frequency), the first term decreases as n^2 and stability is improved ("high-C" circuit).

If L_3' is replaced by L_4 and C_5 in series, (8) becomes

$$\frac{\delta\omega}{\delta V} = \frac{\dfrac{1}{\mu}\dfrac{\partial\mu}{\partial V}\left[\dfrac{X_1 X_2 (X_4 - X_5)}{r_p r_p} + \dfrac{X_2 X_4}{Q_4 r_g}\right]}{-\dfrac{X_1}{\mu X_2}[2L_4]} \tag{11}$$

$$= -\frac{\partial\mu}{\partial V}\left[\frac{1}{2\omega n^2 C_0^2 r_g r_p}\frac{L_3'}{L_4} + \frac{1}{2Q_4 n C_0 r_g}\right]. \tag{12}$$

for $C_1 = C_2 = nC_0$

Note that, for ω = constant, $(X_4 - X_5)$ in the numerator is equal to $X_3 = \omega L_3'$ where L_3' is that value of inductance which would tune to the desired frequency with capacitances C_1 and C_2 only. L_4 can be many times L_3', in which case the effect of the first term is still further reduced in the ratio of L_3'/L_4. The second term s unaltered for the same value of C_2, as long as $Q_4 = Q_3$. This condition may be difficult to meet as L_4 is made larger and larger compared with L_3'. However, a large reduction in the first term can be made, with small increase in the second term in any case.

Summarizing, these results show that in (11) the stability is improved when all reactances in the numerator are kept small, r_g and r_p are kept high, and the rate of change of reactance of the circuit, in the denominator, is made as large as possible. The Q of L_4 should be kept as high as possible.

This class of oscillator has been described by Jefferson[4] as being only "potentially stable," meaning that the change in frequency can only approach zero, but never reach zero, no matter how many elements are used in each branch of the pi network.

Llewellyn[5] has given many circuits with the conditions for frequency stabilization. In practice, however, many of these conditions are modified by the effects of tube capacitances, capacitances of coils, etc., so that either the conditions for zero frequency change are appreciably altered or become critical if the frequency is changed. Also, many of the circuits shown are not readily adaptable to variable-frequency or bandswitching operation.

The result is that for many practical applications an oscillator circuit which can be made to approach perfect stability in a *noncritical* manner, even though perfect stability cannot be achieved, is preferable to a circuit which can be made perfectly stable but only by critical adjustments or by adjustments which must be changed when the frequency is changed.

The long-time stability of this circuit depends almost entirely on the permanence of the elements L_4, C_5, and their temperature coefficients. The variations in frequency caused by temperature changes will generally be found to be much simpler and more straightforward than in conventional circuits because of the fact that the tube effects have been effectively eliminated.

The circuit described above can be set up with reactances of 70 to 100 ohms for X_1 and X_2, these being about one-fortieth of the coil or tuning-capacitance reactance. The circuit has been operated successfully at frequencies ranging from 10 kc. to over 100 Mc. It has been applied in heterodyne frequency meters, oscillators for beat-frequency oscillators, master oscillators in amateur transmitters, and as a frequency-modulation generator. In the latter case, the frequency swing is caused by varying C_1, the center frequency being set by the series tuning capacitor C_5.

The stabilities obtained depend on the frequency and reactances used. Frequency changes of less than 1 part per million to a very few parts per million for changes in supply voltages of ± 15 per cent have been obtained. Interchanging tubes of the same type causes practically no change in frequency.

[4] H. Jefferson, "Stabilization of feedback oscillators," *Wireless Eng.*, vol. 22, pp. 384–389; August, 1945.
[5] F. B. Llewellyn, "Constant frequency oscillators," Proc. I.R.E., vol. 19, pp. 2063–2094; December, 1931.

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